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四杆封闭结构在任意荷载作用下的精确解

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摘要:对任意荷载作用下无侧移的四杆封闭结构给出用分配系数和传递系数表示的精确解.先求四杆封闭结构在任意荷载作用下的各杆的固端弯矩,然后求得结点不平衡力矩,从而将求任意荷载作用的问题转化为求任意的结点不平衡力矩作用的问题.用弯矩分配法求解时,将杆端弯矩表示为无穷多次分配弯矩和传递变矩之和,其表达式为公比小于 1 的无穷等比级数的和,可方便地求得该和的表达式,从而求得任意荷载作用下四杆封闭结构的精确解.文中还给出了算例.

关键词:精确解;刚架;位移法;弯矩分配法

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设图 1 所示四杆封闭结构可忽略侧移的影响.本文欲求其在任意荷载作用下的精确解.用弯矩分配法求解时,将各结点用假设的刚臂固定,求得任意荷载引起的固端弯矩,然后求得结点不平衡力矩,从而将求任意荷载作用的问题转化为求任意的结点不平衡力矩作用的问题.本文以图 1 结点 A 有不平衡力矩 m_a 作用为例,求其精确解.

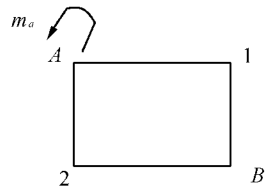

图 1 m_a 作用下四杆封闭结构

Fig.1 Structure subjected to m_a

1 结点 B 用刚臂固定

四杆封闭结构结点 A 有不平衡力矩 m_a 作用.结点 B 用刚臂固定,结点 A,1,2 放松,在结点 A,1,2 同时进行分配和传递,即用集体分配法[1,2],得各杆端弯矩为

$$\left.\begin{aligned} M_{a1} &= (-u_{a1}+g_{a1})m_aG_a & M_{a2} &= (-u_{a2}+g_{a2})m_aG_a \\ M_{1a} &= (-1+u_{1a})c_{a1}u_{a1}m_aG_a & M_{2a} &= (-1+u_{2a})c_{a2}u_{a2}m_aG_a \\ M_{1b} &= u_{1b}c_{a1}u_{a1}m_aG_a & M_{2b} &= u_{2b}c_{a2}u_{a2}m_aG_a \\ M_{b1} &= c_{1b}u_{1b}c_{a1}u_{a1}m_aG_a & M_{b2} &= c_{2b}u_{2b}c_{a2}u_{a2}m_aG_a \end{aligned}\right\} \tag{1}$$

其中

$$g_{a1} = g_{1a} = c_{a1}c_{1a}u_{a1}u_{1a} \qquad g_{a2} = g_{2a} = c_{a2}c_{2a}u_{a2}u_{2a} \tag{2}$$

$$G_a = 1/(1 - g_{a1} - g_{a2}) \tag{3}$$

可见 g_{a1}为杆件 $A1$ 的两端的分配系数 $u_{a1}u_{1a}$和双向传递系数 $c_{a1}c_{1a}$的乘积,我们称为杆件 $A1$ 的性质系数.同理 g_{a2}称为杆件 $A2$ 的性质系数.G_a 与和结点 A 相连的杆件 $A1$ 和 $A2$ 的杆件性质系数 $g_{a1}g_{a2}$有关,我们称之为结点 A 的性质系数.

式(1)可按如下的弯矩分配法得到,在结点 A 对 m_aG_a 进行分配,分配后传递至 1,2 结点,分别在 1 和 2 结点对传递来的弯矩进行分配,分配后传递至 A,B 结点,分配和传递到此结束,叠加分配弯矩和传递弯矩后得各杆端弯矩表达式(1).

式(1)是结点 B 用刚臂固定的精确解,将式(1)的 M_{b1}和 M_{b2}相加得结点 B 的不平衡力矩 m_b,

$$m_b = M_{b1} + M_{b2} = (c_{1b}u_{1b}c_{a1}u_{a1} + c_{2b}u_{2b}c_{a2}u_{a2})m_aG_a \tag{4}$$

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2　结点 A 用刚臂固定

为求解结点 B 在式(4)表示的不平衡力矩 m_b 作用下的解,再将结点 A 用刚臂固定,结点 B,1,2 放松,在结点 B,1,2 同时进行分配和传递,即用集体分配法[1,2],得各杆端弯矩为

$$\left.\begin{array}{ll} M_{b1} = (-u_{b1} + g_{b1})m_bG_b & M_{b2} = (-u_{b2} + g_{b2})m_bG_b \\ M_{1b} = (-1 + u_{1b})c_{b1}u_{b1}m_bG_b & M_{2b} = (-1 + u_{2b})c_{b2}u_{b2}m_bG_b \\ M_{1a} = u_{1a}c_{b1}u_{b1}m_bG_b & M_{2a} = u_{2a}c_{b2}u_{b2}m_bG_b \\ M_{a1} = c_{1a}u_{1a}c_{b1}u_{b1}m_bG_b & M_{a2} = c_{2a}u_{2a}c_{b2}u_{b2}m_bG_b \end{array}\right\} \tag{5}$$

其中

$$g_{b1} = g_{1b} = c_{b1}c_{1b}u_{b1}u_{1b} \qquad g_{b2} = g_{2b} = c_{b2}c_{2b}u_{b2}u_{2b} \tag{6}$$

$$G_b = 1/(1 - g_{b1} - g_{b2}) \tag{7}$$

可见 g_{b1}为杆件 $B1$ 的两端的分配系数 $u_{b1}u_{1b}$和双向传递系数$c_{b1}c_{1b}$的乘积,称为杆件 $B1$ 的性质系数.同理 g_{b2}称为杆件 $B2$ 的性质系数.G_b 与和结点 B 相连的杆件 $B1$ 和 $B2$ 的杆件性质系数 $g_{b1}g_{b2}$有关,称为结点 B 的性质系数.

式(5)是结点 A 用刚臂固定时的精确解,将式(5)的 M_{a1}和 M_{a2}相加得 A 点的不平衡力矩m_a^1,

$$m_a^1 = M_{a1} + M_{a2} = (c_{1a}u_{1a}c_{b1}u_{b1} + c_{2a}u_{2a}c_{b2}u_{b2})m_bG_b \tag{8}$$

将式(4)代入上式得

$$m_a^1 = (c_{1a}u_{1a}c_{b1}u_{b1} + c_{2a}u_{2a}c_{b2}u_{b2})(c_{1b}u_{1b}c_{a1}u_{a1} + c_{2b}u_{2b}c_{a2}u_{a2})m_aG_aG_b \tag{9}$$

到此,完成了一个循环.

3　无穷多次循环

进行无穷多次循环,循环中结点 A 的不平衡力矩依次为 m_a,m_a^1,m_a^2,m_a^3,m_a^4,…,其和用 H_a 表示,循环中结点 B 的不平衡力矩依次为m_b,m_b^1,m_b^2,m_b^3,m_b^4,…,其和用 H_b 表示,即

$$H_a = m_a + m_a^1 + m_a^2 + m_a^3 + \cdots \tag{10}$$

$$H_b = m_b + m_b^1 + m_b^2 + m_b^3 + \cdots \tag{11}$$

显而易见,可将式(9)推广,即将式(9)左边的 m_a^1 改为 m_a^{j+1},右边的 m_a 改为 m_a^j 并两边除以m_a^j,即得

$$m_a^{j+1}/m_a^j = (c_{1a}u_{1a}c_{b1}u_{b1} + c_{2a}u_{2a}c_{b2}u_{b2})(c_{1b}u_{1b}c_{a1}u_{a1} + c_{2b}u_{2b}c_{a2}u_{a2})G_aG_b \tag{12}$$

从物理概念可知,上式比值小于 1.上式比值为式(10)中的无穷等比级数的公比,故式(10)为一公比小于 1 的无穷等比级数的和:

$$H_a = m_a/[1 - (c_{1a}u_{1a}c_{b1}u_{b1} + c_{2a}u_{2a}c_{b2}u_{b2})(c_{1b}u_{1b}c_{a1}u_{a1} + c_{2b}u_{2b}c_{a2}u_{a2})G_aG_b] \tag{13}$$

显而易见,可将式(4)推广,即将式(4)左边的 m_b 改为 m_b^j,右边的 m_a 改为 m_a^j,即:

$$m_b^j = (c_{1b}u_{1b}c_{a1}u_{a1} + c_{2b}u_{2b}c_{a2}u_{a2})m_a^jG_a \qquad j = 1,2,3,\cdots \tag{14}$$

将式(4)(14)代入式(11)得

$$H_b = (c_{1b}u_{1b}c_{a1}u_{a1} + c_{2b}u_{2b}c_{a2}u_{a2})G_a(m_a + m_a^1 + m_a^2 + m_a^3 + \cdots) \tag{15}$$

显而易见,由式(10),上式可表示为

$$H_b = (c_{1b}u_{1b}c_{a1}u_{a1} + c_{2b}u_{2b}c_{a2}u_{a2})G_aH_a \tag{16}$$

4　在结点 A 不平衡力矩 m_a 作用下的精确解

结点 A 在不平衡力矩H_a 作用下,将结点 B 用刚臂固定,结点 A,1,2 放松,如图 2 所示,在结点 A,1,2 同时进行分配和传递,得各杆端弯矩,即式(1)中 m_a 改为 H_a:

$$\left.\begin{array}{ll} M_{a1} = (-u_{a1} + g_{a1})H_aG_a & M_{a2} = (-u_{a2} + g_{a2})H_aG_a \\ M_{1a} = (-1 + u_{1a})c_{a1}u_{a1}H_aG_a & M_{2a} = (-1 + u_{2a})c_{a2}u_{a2}H_aG_a \\ M_{1b} = u_{1b}c_{a1}u_{a1}H_aG_a & M_{2b} = u_{2b}c_{a2}u_{a2}H_aG_a \\ M_{b1} = c_{1b}u_{1b}c_{a1}u_{a1}H_aG_a & M_{b2} = c_{2b}u_{2b}c_{a2}u_{a2}H_aG_a \end{array}\right\} \tag{17}$$

结点 B 在不平衡力矩 H_b 作用下,将结点 B 用刚臂固定,结点 A,1,2 放松,如图 3 所示,在结点 A,1,2 同时进行分配和传递,得各杆端弯矩,即式(5)中 m_b 改为 H_b:

$$\left.\begin{array}{ll} M_{b1} = (-u_{b1} + g_{b1}) H_b G_b & M_{b2} = (-u_{b2} + g_{b2}) H_b G_b \\ M_{1b} = (-1 + u_{1b}) c_{b1} u_{b1} H_b G_b & M_{2b} = (-1 + u_{2b}) c_{b2} u_{b2} H_b G_b \\ M_{1a} = u_{1a} c_{b1} u_{b1} H_b G_b & M_{2a} = u_{2a} c_{b2} u_{b2} H_b G_b \\ M_{a1} = c_{1a} u_{1a} c_{b1} u_{b1} H_b G_b & M_{a2} = c_{2a} u_{2a} c_{b2} u_{b2} H_b G_b \end{array}\right\} \tag{18}$$

图 1 四杆封闭结构在结点 A 不平衡力矩 m_a 作用下用分配系数和传递系数表示的精确解为式(17)和(18)的叠加.其中 H_a 由式(13)表示,H_b 由式(16)表示,杆件性质系数和结点性质系数由式(2)(3)(6)(7)表示.

5 算 例

设图 1 各分配系数和传递系数均为 $\frac{1}{2}$,求在 m_a 作用下各杆端弯矩.

由式(2)(6)计算各杆性质系数:

$$g_{a1} = g_{a2} = g_{b1} = g_{b2} = \frac{1}{16}$$

由式(3)(7)计算各结点性质系数:

$$G_a = G_b = \frac{8}{7}$$

由式(13)计算 H_a:

$$H_a = \frac{m_a}{\left[1 - \frac{1}{8} \times \frac{1}{8} \times \frac{8}{7} \times \frac{8}{7}\right]} = \frac{m_a 49}{48}$$

由式(16)计算 H_b:

$$H_b = \frac{1}{8} \times \frac{8}{7} \times \frac{m_a 49}{48} = \frac{m_a 7}{48}$$

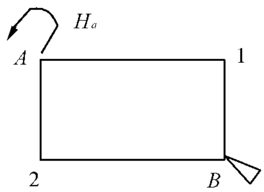

图 2 H_a 作用下四杆封闭结构

Fig.2 Structure subjected to H_a

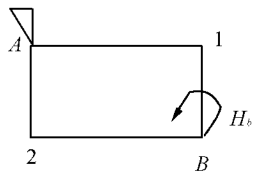

图 3 H_b 作用下四杆封闭结构

Fig.3 Structure subjected to H_b

由式(17)计算图 2 在 $H_a = \frac{m_a 49}{48}$ 作用下各杆端弯矩:

$$m_{a1} = m_{a2} = \left(-\frac{1}{2} + \frac{1}{16}\right) \times \frac{49}{48} \times \frac{8}{7} \times m_a = \frac{-48 m_a}{96}$$

$$m_{1a} = m_{2a} = \frac{-1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{49}{48} \times \frac{8}{7} \times m_a = -\frac{7 m_a}{48}$$

$$m_{1b} = m_{2b} = \frac{1}{8} \times \frac{49}{48} \times \frac{8}{7} \times m_a = \frac{7 m_a}{48}$$

$$m_{b1} = m_{b2} = \frac{1}{16} \times \frac{49}{48} \times \frac{8}{7} \times m_a = \frac{7 m_a}{96}$$

由式(18)计算图 3 在 $H_b = \frac{m_a 7}{48}$ 作用下各杆端弯矩:

$$m_{b1} = m_{b2} = \left(\frac{-1}{2} + \frac{1}{16}\right) \times \frac{7}{48} \times \frac{8}{7} \times m_a = \frac{-7 m_a}{96}$$

$$m_{1b} = m_{2b} = \frac{-1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{7}{48} \times \frac{8}{7} \times m_a = -\frac{m_a}{48}$$

$$m_{1a} = m_{2a} = \frac{1}{8} \times \frac{7}{48} \times \frac{8}{7} \times m_a = \frac{m_a}{48}$$

$$m_{a1} = m_{a2} = \frac{1}{16} \times \frac{7}{48} \times \frac{8}{7} \times m_a = \frac{m_a}{96}$$

将上述图 2、图 3 的各杆端弯矩分别叠加即得欲求的解

$$m_{a1} = m_{a2} = \left(-\frac{49}{96} + \frac{1}{96}\right) m_a = -\frac{m_a}{2}$$

$$m_{1a} = m_{2a} = \left(-\frac{7}{8} + \frac{1}{48}\right) m_a = -\frac{m_a}{8}$$

$$m_{1b} = m_{2b} = \left(-\frac{7}{48} - \frac{1}{48}\right) m_a = -\frac{m_a}{8}$$

$$m_{b1} = m_{b2} = \left(\frac{7}{96} - \frac{7}{96}\right) m_a = 0$$

6 结　　语

本文对任意荷载作用下无侧移四杆封闭结构给出用分配系数和传递系数表示的精确解,有一定的理论意义和实用意义.

参考文献:

[1] 武汉水利电力学院建筑力学教研室.结构力学(第 1 版)[M].北京:水利电力出版社,1960.197~206.

[2] 徐汉忠.结构力学中结点不平衡力矩分配传递的极限公式[J].河海大学学报,1997,25(6):60~64.

[3] Jeffrey P Laible.Structural analysis[M].Holt,Rrinehart and Winston:CBS College Publishing,1950.536~563.

Analytic Solution for a Closed Structure with Four Bars Subjected to Arbitrary Loads

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Abstract: Presented is an analytic solution for a closed structure with four bars subjected to arbitrary loads, which is expressed by the distribution factors and carried over factors. The fixed end bending moments are calculated, and then the unbalanced joint moments are obtained, thus the structure subjected to the arbitrary unbalanced joint moments instead of arbitrary loads is studied. The total moment on the ends is the sum of all the distributed moments and the carried over end moments, and can be expressed as the sum of an infinite series with a common ratio smaller than one. Thus the analytic solution is found conveniently. A numerical example is also presented in this paper.

Key words: analytic solution; rigid frames; displacement method; bending moment distribution method