三向荷载作用下液固耦合平面有限元分析

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摘要:根据广义平面应变问题具有所有物理量沿某一方向变化为零的特点,针对连续、各向异性两相饱和介质,在变形满足小变形、孔压流满足达西定律、有效应力原理成立的假定下,建立以增量形式的三向荷载作用下液固耦合作用的平面有限元分析方法.

关键词:液固耦合;三向荷载;平面应变;有限元

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无论在岩体结构还是在土体结构中,液固两相耦合问题是一个十分普遍而又重要的问题,如岩体渗流与应力的耦合问题、土体的固结问题等.由于这类问题的复杂性,除极简单情况,欲寻求其解析解往往是十分困难的,因此人们常常借助某种数值手段来解决,其中有限元方法是一种十分有用的手段.自 1941 年 Biot 提出著名的三维固结理论之后,许多学者对该类问题的数值解法进行了研究,已取得了较丰硕的成果[1~4].笔者也曾对岩体渗流与应力耦合问题的三维有限元解法进行了研究,取得了较满意的结果.然而,在工程实际中,往往会遇到结构为二维但荷载为三维的情况,如断面形状和尺寸沿某一方向不变,但沿该方向有荷载作用的情况.此类问题不属于常规的平面应变问题,但如果采用三维有限元法求解,又使问题人为地复杂化,很不经济.本文专门研究此类结构为二维、荷载为三维的有限元解法,并考虑了液相的可压缩性.

1　广义平面应变问题的特点

广义平面应变问题的特点是各种物理量如水压力、位移、应力、荷载等沿某一方向的变化量为零.

2　液固两相耦合广义平面应变问题的基本方程

假定介质为连续的、各向异性的液固两相饱和体,变形满足小变形协调定律,孔压流满足达西定律,并假定有效应力原理成立,则根据广义平面应变问题的特征(假定各物理量沿 y 方向的变化量为零),可以建立如下方程:

2.1　平衡方程

根据单元体力的平衡条件,可建立

$$\begin{cases}\Delta\sigma'_{11,1}+\Delta\sigma_{31,3}+\Delta u_{w,1}-\rho\Delta f_1=0\\ \Delta\sigma_{12,1}+\Delta\sigma_{32,3}-\rho\Delta f_2=0\\ \Delta\sigma_{13,1}+\Delta\sigma'_{33,3}+\Delta u_{w,3}-\rho\Delta f_3=0\end{cases}\tag{1}$$

式中:$\rho\Delta f_i$——i 方向的体力分量增量;Δu_w——孔隙水压力增量;$\Delta\sigma_{ij}'$——有效应力分量增量.下标 1,2,3 分别表示 x,y,z 坐标;下标‘,1’和下标‘,3’分别表示对坐标 x 和 z 的偏导数.

根据单元体流量平衡条件,可建立

$$k_{11}^{*}(\rho_w\Delta f_{1,1}-\Delta u_{w,11})+k_{33}^{*}(\rho_w\Delta f_{3,3}-\Delta u_{w,33})+n\Delta\dot{\varepsilon}_{vwc}=\Delta\dot{\varepsilon}_v\tag{2}$$

其中

$$k_{ij}^{*}=k_{ij}/\rho_w g\tag{3}$$

式中:$\dot{\varepsilon}$——ε 对时间求偏导;$\Delta\varepsilon_v$——固相骨架变形引起的体变增量;$\Delta\varepsilon_{vwc}$——液相压缩引起的液相体变增

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量;k_{ij}^{*}——有效渗透张量元素;k_{ij}——渗透张量元素;ρ_w——水的密度;g——重力加速度.

2.2 本构方程

a. 固相骨架应力应变关系

$$\begin{Bmatrix} \Delta\sigma'_{11} \\ \Delta\sigma'_{22} \\ \Delta\sigma'_{33} \\ \Delta\sigma_{12} \\ \Delta\sigma_{13} \\ \Delta\sigma_{23} \end{Bmatrix} = \begin{bmatrix} d_{11} & d_{12} & d_{13} & d_{14} & d_{15} & d_{16} \\ d_{21} & d_{22} & d_{23} & d_{24} & d_{25} & d_{26} \\ d_{31} & d_{32} & d_{33} & d_{34} & d_{35} & d_{36} \\ d_{41} & d_{42} & d_{43} & d_{44} & d_{45} & d_{46} \\ d_{51} & d_{52} & d_{53} & d_{54} & d_{55} & d_{56} \\ d_{61} & d_{62} & d_{63} & d_{64} & d_{65} & d_{66} \end{bmatrix} \begin{Bmatrix} \Delta\varepsilon_{11} \\ \Delta\varepsilon_{22} \\ \Delta\varepsilon_{33} \\ 2\Delta\varepsilon_{12} \\ 2\Delta\varepsilon_{13} \\ 2\Delta\varepsilon_{23} \end{Bmatrix} + \begin{Bmatrix} \eta_c\Delta\dot{\varepsilon}_{11} \\ \eta_c\Delta\dot{\varepsilon}_{22} \\ \eta_c\Delta\dot{\varepsilon}_{33} \\ \eta_s 2\Delta\dot{\varepsilon}_{12} \\ \eta_s 2\Delta\dot{\varepsilon}_{13} \\ \eta_s 2\Delta\dot{\varepsilon}_{23} \end{Bmatrix} \tag{4}$$

式中:d_{ij}——应力应变刚度矩阵元素;η_c,η_s——压缩阻尼系数和剪切阻尼系数;$\Delta\varepsilon_{ij}$——应变分量的增量.根据广义平面应变问题的特点,任一变量对 y 方向的偏导数为零,因此有

$$\begin{Bmatrix} \Delta\varepsilon_{11} \\ \Delta\varepsilon_{22} \\ \Delta\varepsilon_{33} \\ 2\Delta\varepsilon_{12} \\ 2\Delta\varepsilon_{13} \\ 2\Delta\varepsilon_{23} \end{Bmatrix} = \begin{Bmatrix} -\Delta u_{1,1} \\ 0 \\ -\Delta u_{3,3} \\ -\Delta u_{2,1} \\ -(\Delta u_{1,3}+\Delta u_{3,1}) \\ -\Delta u_{2,3} \end{Bmatrix} \tag{5}$$

b. 液相水压力体变关系

假设水的体积模量为 Γ_w,则有

$$\Delta u_w = \Gamma_w \cdot \Delta\varepsilon_{vwc} \tag{6}$$

3 有限元支配方程的建立和求解

将式(4)(5)(6)代入式(1)和式(2),可建立以位移和孔隙水压力表示的平衡方程,再应用加权余量法,可建立如下对任意权函数 v 成立的积分方程组:

$$\left\{\begin{aligned}
&\iint_\Omega (-d_{11}\Delta u_{1,11} - d_{13}\Delta u_{3,31} - d_{14}\Delta u_{2,11} - d_{15}\Delta u_{1,31} - d_{15}\Delta u_{3,11} - d_{16}\Delta u_{2,31} - \\
&\quad d_{51}\Delta u_{1,13} - d_{53}\Delta u_{3,33} - d_{54}\Delta u_{2,13} - d_{55}\Delta u_{1,33} - d_{55}\Delta u_{3,13} - d_{56}\Delta u_{2,33} + \Delta u_{w,1})\, v\,\mathrm{d}x_1\mathrm{d}x_3 + \\
&\iint_\Omega (-\eta_c\Delta\dot{u}_{1,11} - \eta_s\Delta\dot{u}_{1,33} - \eta_s\Delta\dot{u}_{3,13})\, v\,\mathrm{d}x_1\mathrm{d}x_3 = \iint_\Omega \rho\Delta f_1 v\,\mathrm{d}x_1\mathrm{d}x_3 \\
&\iint_\Omega (-d_{41}\Delta u_{1,11} - d_{43}\Delta u_{3,31} - d_{44}\Delta u_{2,11} - d_{45}\Delta u_{1,31} - d_{45}\Delta u_{3,11} - d_{46}\Delta u_{2,31} - \\
&\quad d_{61}\Delta u_{1,13} - d_{63}\Delta u_{3,33} - d_{64}\Delta u_{2,13} - d_{65}\Delta u_{1,33} - d_{65}\Delta u_{3,13} - d_{66}\Delta u_{2,33})\, v\,\mathrm{d}x_1\mathrm{d}x_3 + \\
&\iint_\Omega (-\eta_s\Delta\dot{u}_{2,11} - \eta_s\Delta\dot{u}_{2,33})\, v\,\mathrm{d}x_1\mathrm{d}x_3 = \iint_\Omega \rho\Delta f_2 v\,\mathrm{d}x_1\mathrm{d}x_3 \\
&\iint_\Omega (-d_{51}\Delta u_{1,11} - d_{53}\Delta u_{3,31} - d_{54}\Delta u_{2,11} - d_{55}\Delta u_{1,31} - d_{55}\Delta u_{3,11} - d_{56}\Delta u_{2,31} - \\
&\quad d_{31}\Delta u_{1,13} - d_{33}\Delta u_{3,33} - d_{34}\Delta u_{2,13} - d_{35}\Delta u_{1,33} - d_{35}\Delta u_{3,13} - d_{36}\Delta u_{2,33} + \Delta u_{w,3})\, v\,\mathrm{d}x_1\mathrm{d}x_3 + \\
&\iint_\Omega (-\eta_s\Delta\dot{u}_{1,31} - \eta_s\Delta\dot{u}_{3,11} - \eta_c\Delta\dot{u}_{3,33})\, v\,\mathrm{d}x_1\mathrm{d}x_3 = \iint_\Omega \rho\Delta f_3 v\,\mathrm{d}x_1\mathrm{d}x_3 \\
&\iint_\Omega (\Delta\dot{u}_{1,1} + \Delta\dot{u}_{3,3} + \Delta\dot{u}_w\Gamma^{-1} - k_{11}^{*}\,\Delta u_{w,11} - k_{33}^{*}\,\Delta u_{w,33})\, v\,\mathrm{d}x_1\mathrm{d}x_3 = \\
&-\iint_\Omega (k_{11}^{*}\,\rho_w\Delta f_{1,1} + k_{33}^{*}\,\rho_w\Delta f_{3,3})\, v\,\mathrm{d}x_1\mathrm{d}x_3
\end{aligned}\right. \tag{7}$$

式中:Γ——土体的体积模量,$\Gamma = \Gamma_w / n$,n 为孔隙率.

对式(7)进行分部积分,并考虑边界条件,则可以建立:

$$\begin{aligned}
&\iint_\Omega (\eta_c \Delta \dot{u}_{1,1} v_{,1} + \eta_s \Delta \dot{u}_{1,3} v_{,3}) \mathrm{d}x_1 \mathrm{d}x_3 + \iint_\Omega (\eta_s \Delta \dot{u}_{3,1} v_{,3}) \mathrm{d}x_1 \mathrm{d}x_3 + \\
&\iint_\Omega [(d_{11}\Delta u_{1,1} + d_{15}\Delta u_{1,3}) v_{,1} + (d_{51}\Delta u_{1,1} + d_{55}\Delta u_{1,3}) v_{,3}] \mathrm{d}x_1 \mathrm{d}x_3 + \\
&\iint_\Omega [(d_{14}\Delta u_{2,1} + d_{16}\Delta u_{2,3}) v_{,1} + (d_{54}\Delta u_{2,1} + d_{56}\Delta u_{2,3}) v_{,3}] \mathrm{d}x_1 \mathrm{d}x_3 + \\
&\iint_\Omega [(d_{13}\Delta u_{3,3} + d_{15}\Delta u_{3,1}) v_{,1} + (d_{53}\Delta u_{3,3} + d_{55}\Delta u_{3,1}) v_{,3}] \mathrm{d}x_1 \mathrm{d}x_3 - \iint_\Omega \Delta u_w v_{,1} \mathrm{d}x_1 \mathrm{d}x_3 = \\
&\iint_\Omega \rho \Delta f_1 v \mathrm{d}x_1 \mathrm{d}x_3 + \int_{\partial \Omega_\sigma} \Delta \hat{T}_1 v \mathrm{d}s
\end{aligned} \tag{8}$$

$$\begin{aligned}
&\iint_\Omega (\eta_s \Delta \dot{u}_{2,1} v_{,1} + \eta_s \Delta \dot{u}_{2,3} v_{,3}) \mathrm{d}x_1 \mathrm{d}x_3 + \\
&\iint_\Omega [(d_{41}\Delta u_{1,1} + d_{45}\Delta u_{1,3}) v_{,1} + (d_{61}\Delta u_{1,1} + d_{65}\Delta u_{1,3}) v_{,3}] \mathrm{d}x_1 \mathrm{d}x_3 + \\
&\iint_\Omega [(d_{44}\Delta u_{2,1} + d_{46}\Delta u_{2,3}) v_{,1} + (d_{64}\Delta u_{2,1} + d_{66}\Delta u_{2,3}) v_{,3}] \mathrm{d}x_1 \mathrm{d}x_3 + \\
&\iint_\Omega [(d_{43}\Delta u_{3,3} + d_{45}\Delta u_{3,1}) v_{,1} + (d_{63}\Delta u_{3,3} + d_{65}\Delta u_{3,1}) v_{,3}] \mathrm{d}x_1 \mathrm{d}x_3 = \\
&\iint_\Omega \rho \Delta f_2 v \mathrm{d}x_1 \mathrm{d}x_3 + \int_{\partial \Omega_\sigma} \Delta \hat{T}_2 v \mathrm{d}s
\end{aligned} \tag{9}$$

$$\begin{aligned}
&\iint_\Omega (\eta_s \Delta \dot{u}_{1,3} v_{,1}) \mathrm{d}x_1 \mathrm{d}x_3 + \iint_\Omega (\eta_s \Delta \dot{u}_{3,1} v_{,1} + \eta_c \Delta \dot{u}_{3,3} v_{,3}) \mathrm{d}x_1 \mathrm{d}x_3 + \\
&\iint_\Omega [(d_{51}\Delta u_{1,1} + d_{55}\Delta u_{1,3}) v_{,1} + (d_{31}\Delta u_{1,1} + d_{35}\Delta u_{1,3}) v_{,3}] \mathrm{d}x_1 \mathrm{d}x_3 + \\
&\iint_\Omega [(d_{54}\Delta u_{2,1} + d_{56}\Delta u_{2,3}) v_{,1} + (d_{34}\Delta u_{2,1} + d_{36}\Delta u_{2,3}) v_{,3}] \mathrm{d}x_1 \mathrm{d}x_3 + \\
&\iint_\Omega [(d_{53}\Delta u_{3,3} + d_{55}\Delta u_{3,1}) v_{,1} + (d_{33}\Delta u_{3,3} + d_{35}\Delta u_{3,1}) v_{,3}] \mathrm{d}x_1 \mathrm{d}x_3 - \iint_\Omega \Delta u_w v_{,3} \mathrm{d}x_1 \mathrm{d}x_3 = \\
&\iint_\Omega \rho \Delta f_3 v \mathrm{d}x_1 \mathrm{d}x_3 + \int_{\partial \Omega_\sigma} \Delta \hat{T}_3 v \mathrm{d}s
\end{aligned} \tag{10}$$

$$\begin{aligned}
&\iint_\Omega \Delta \dot{u}_{1,1} v \mathrm{d}x_1 \mathrm{d}x_3 + \iint_\Omega \Delta \dot{u}_{3,3} v \mathrm{d}x_1 \mathrm{d}x_3 + \iint_\Omega \Delta \dot{u}_w \Gamma^{-1} v \mathrm{d}x_1 \mathrm{d}x_3 + \\
&\iint_\Omega k_{11}^* \Delta u_{w,1} v_{,1} \mathrm{d}x_1 \mathrm{d}x_3 + \iint_\Omega k_{33}^* \Delta u_{w,3} v_{,3} \mathrm{d}x_1 \mathrm{d}x_3 = \\
&\iint_\Omega (k_{11}^* \rho_w \Delta f_1 v_{,1} + k_{33}^* \rho_w \Delta f_3 v_{,3}) \mathrm{d}x_1 \mathrm{d}x_3 + \int_{\partial \Omega_q} \Delta \hat{q}_n v \mathrm{d}s
\end{aligned} \tag{11}$$

式中:$\Delta \hat{T}_j$——应力边界 j 方向面力分量增量;$\Delta \hat{q}_n$——流量边界法向流量增量.

以形函数为权函数,采用伽辽金有限元法[5],在空间上对位移和孔压离散,即 $\Delta u \approx \sum_{j=1}^{N} \phi_j \Delta u_j$,则可以建立如下的有限元支配方程:

$$
\begin{bmatrix} \boldsymbol{C}_{11}^k & \boldsymbol{C}_{12}^k & \cdots & \boldsymbol{C}_{1n}^k \\ \boldsymbol{C}_{21}^k & \boldsymbol{C}_{22}^k & \cdots & \boldsymbol{C}_{2n}^k \\ \vdots & \vdots & & \vdots \\ \boldsymbol{C}_{n1}^k & \boldsymbol{C}_{n2}^k & \cdots & \boldsymbol{C}_{nn}^k \end{bmatrix} \begin{Bmatrix} \Delta\dot{\boldsymbol{A}}_1 \\ \Delta\dot{\boldsymbol{A}}_2 \\ \vdots \\ \Delta\dot{\boldsymbol{A}}_n \end{Bmatrix} + \begin{bmatrix} \boldsymbol{S}_{11}^k & \boldsymbol{S}_{12}^k & \cdots & \boldsymbol{S}_{1n}^k \\ \boldsymbol{S}_{21}^k & \boldsymbol{S}_{22}^k & \cdots & \boldsymbol{S}_{2n}^k \\ \vdots & \vdots & & \vdots \\ \boldsymbol{S}_{n1}^k & \boldsymbol{S}_{n2}^k & \cdots & \boldsymbol{S}_{nn}^k \end{bmatrix} \begin{Bmatrix} \Delta\boldsymbol{A}_1 \\ \Delta\boldsymbol{A}_2 \\ \vdots \\ \Delta\boldsymbol{A}_n \end{Bmatrix} = \begin{Bmatrix} \Delta\boldsymbol{F}_1^k \\ \Delta\boldsymbol{F}_2^k \\ \vdots \\ \Delta\boldsymbol{F}_n^k \end{Bmatrix} \tag{12}
$$

式中

$$
\Delta\boldsymbol{A}_j = \{\Delta u_{1j}, \Delta u_{2j}, \Delta u_{3j}, \Delta u_{wj}\}^{\mathrm{T}} \qquad \Delta\dot{\boldsymbol{A}}_j = \{\Delta\dot{u}_{1j}, \Delta\dot{u}_{2j}, \Delta\dot{u}_{3j}, \Delta\dot{u}_{wj}\}^{\mathrm{T}}
$$

$$
\boldsymbol{C}_{ij}^k = \begin{bmatrix} C_{11}^{k,ij} & 0 & C_{13}^{k,ij} & 0 \\ 0 & C_{22}^{k,ij} & 0 & 0 \\ C_{31}^{k,ij} & 0 & C_{33}^{k,ij} & 0 \\ C_{41}^{k,ij} & 0 & C_{43}^{k,ij} & C_{44}^{k,ij} \end{bmatrix} \quad \boldsymbol{S}_{ij}^k = \begin{bmatrix} S_{11}^{k,ij} & S_{12}^{k,ij} & S_{13}^{k,ij} & S_{14}^{k,ij} \\ S_{21}^{k,ij} & S_{22}^{k,ij} & S_{23}^{k,ij} & 0 \\ S_{31}^{k,ij} & S_{32}^{k,ij} & S_{33}^{k,ij} & S_{34}^{k,ij} \\ 0 & 0 & 0 & S_{44}^{k,ij} \end{bmatrix} \quad \Delta\boldsymbol{F}_i^k = \begin{Bmatrix} \Delta F_1^{k,i} \\ \Delta F_2^{k,i} \\ \Delta F_3^{k,i} \\ \Delta F_4^{k,i} \end{Bmatrix}
$$

其中

$$
C_{11}^{k,ij} = \iint_{\Omega^k} (\eta_c \phi_{i,1}\phi_{j,1} + \eta_s \phi_{i,3}\phi_{j,3})\,\mathrm{d}x_1\mathrm{d}x_3 \qquad C_{31}^{k,ij} = \iint_{\Omega^k} (\eta_s \phi_{i,1}\phi_{j,3})\,\mathrm{d}x_1\mathrm{d}x_3
$$

$$
C_{22}^{k,ij} = \iint_{\Omega^k} (\eta_s \phi_{i,1}\phi_{j,1} + \eta_s \phi_{i,3}\phi_{j,3})\,\mathrm{d}x_1\mathrm{d}x_3 \qquad C_{41}^{k,ij} = \iint_{\Omega^k} \phi_i \phi_{j,1}\,\mathrm{d}x_1\mathrm{d}x_3
$$

$$
C_{33}^{k,ij} = \iint_{\Omega^k} (\eta_s \phi_{i,1}\phi_{j,1} + \eta_c \phi_{i,3}\phi_{j,3})\,\mathrm{d}x_1\mathrm{d}x_3 \qquad C_{43}^{k,ij} = \iint_{\Omega^k} \phi_i \phi_{j,3}\,\mathrm{d}x_1\mathrm{d}x_3
$$

$$
C_{13}^{k,ij} = \iint_{\Omega^k} (\eta_s \phi_{i,3}\phi_{j,1})\,\mathrm{d}x_1\mathrm{d}x_3 \qquad C_{44}^{k,ij} = \iint_{\Omega^k} \Gamma^{-1}\phi_i\phi_j\,\mathrm{d}x_1\mathrm{d}x_3
$$

$$
S_{11}^{k,ij} = \iint_{\Omega^k} (d_{11}\phi_{i,1}\phi_{j,1} + d_{15}\phi_{i,1}\phi_{j,3} + d_{51}\phi_{i,3}\phi_{j,1} + d_{55}\phi_{i,3}\phi_{j,3})\,\mathrm{d}x_1\mathrm{d}x_3
$$

$$
S_{12}^{k,ij} = \iint_{\Omega^k} (d_{14}\phi_{i,1}\phi_{j,1} + d_{16}\phi_{i,1}\phi_{j,3} + d_{54}\phi_{i,3}\phi_{j,1} + d_{56}\phi_{i,3}\phi_{j,3})\,\mathrm{d}x_1\mathrm{d}x_3
$$

$$
S_{13}^{k,ij} = \iint_{\Omega^k} (d_{15}\phi_{i,1}\phi_{j,1} + d_{13}\phi_{i,1}\phi_{j,3} + d_{55}\phi_{i,3}\phi_{j,1} + d_{53}\phi_{i,3}\phi_{j,3})\,\mathrm{d}x_1\mathrm{d}x_3
$$

$$
S_{21}^{k,ij} = \iint_{\Omega^k} (d_{41}\phi_{i,1}\phi_{j,1} + d_{45}\phi_{i,1}\phi_{j,3} + d_{61}\phi_{i,3}\phi_{j,1} + d_{65}\phi_{i,3}\phi_{j,3})\,\mathrm{d}x_1\mathrm{d}x_3
$$

$$
S_{22}^{k,ij} = \iint_{\Omega^k} (d_{44}\phi_{i,1}\phi_{j,1} + d_{46}\phi_{i,1}\phi_{j,3} + d_{64}\phi_{i,3}\phi_{j,1} + d_{66}\phi_{i,3}\phi_{j,3})\,\mathrm{d}x_1\mathrm{d}x_3
$$

$$
S_{23}^{k,ij} = \iint_{\Omega^k} (d_{45}\phi_{i,1}\phi_{j,1} + d_{43}\phi_{i,1}\phi_{j,3} + d_{65}\phi_{i,3}\phi_{j,1} + d_{63}\phi_{i,3}\phi_{j,3})\,\mathrm{d}x_1\mathrm{d}x_3
$$

$$
S_{31}^{k,ij} = \iint_{\Omega^k} (d_{51}\phi_{i,1}\phi_{j,1} + d_{55}\phi_{i,1}\phi_{j,3} + d_{31}\phi_{i,3}\phi_{j,1} + d_{35}\phi_{i,3}\phi_{j,3})\,\mathrm{d}x_1\mathrm{d}x_3
$$

$$
S_{32}^{k,ij} = \iint_{\Omega^k} (d_{54}\phi_{i,1}\phi_{j,1} + d_{56}\phi_{i,1}\phi_{j,3} + d_{34}\phi_{i,3}\phi_{j,1} + d_{36}\phi_{i,3}\phi_{j,3})\,\mathrm{d}x_1\mathrm{d}x_3
$$

$$
S_{33}^{k,ij} = \iint_{\Omega^k} (d_{55}\phi_{i,1}\phi_{j,1} + d_{53}\phi_{i,1}\phi_{j,3} + d_{35}\phi_{i,3}\phi_{j,1} + d_{33}\phi_{i,3}\phi_{j,3})\,\mathrm{d}x_1\mathrm{d}x_3
$$

$$
S_{14}^{k,ij} = -\iint_{\Omega^k} \phi_{i,1}\phi_j\,\mathrm{d}x_1\mathrm{d}x_3 \qquad S_{34}^{k,ij} = -\iint_{\Omega^k} \phi_{i,3}\phi_j\,\mathrm{d}x_1\mathrm{d}x_3
$$

$$
S_{44}^{k,ij} = \iint_{\Omega^k} (k_{11}^{*}\phi_{i,1}\phi_{j,1} + k_{33}^{*}\phi_{i,3}\phi_{j,3})\,\mathrm{d}x_1\mathrm{d}x_3
$$

$$\Delta F_1^{k,i} = \iint_{\Omega^k} \rho \Delta f_1 \phi_i \mathrm{d}x_1 \mathrm{d}x_3 + \int_{\partial \Omega_\sigma} \Delta \hat{T}_1 \phi_i \mathrm{d}s \qquad \Delta F_2^{k,i} = \iint_{\Omega^k} \rho \Delta f_2 \phi_i \mathrm{d}x_1 \mathrm{d}x_3 + \int_{\partial \Omega_\sigma} \Delta \hat{T}_2 \phi_i \mathrm{d}s$$

$$\Delta F_3^{k,i} = \iint_{\Omega^k} \rho \Delta f_3 \phi_i \mathrm{d}x_1 \mathrm{d}x_3 + \int_{\partial \Omega_\sigma} \Delta \hat{T}_3 \phi_i \mathrm{d}s$$

$$\Delta F_4^{k,i} = \iint_{\Omega^k} (k_{11}^{*} \rho_w \Delta f_1 \phi_{i,1} + k_{33}^{*} \rho_w \Delta f_3 \phi_{i,3}) \mathrm{d}x_1 \mathrm{d}x_3 + \int_{\partial \Omega_q} \Delta \hat{q}_n \phi_i \mathrm{d}s$$

将方程(12)整体组装，采用 Newmark-β 方法[5]进行时间离散，则可进行方程组的求解，从而得到 ΔA 和 $\Delta \dot{A}$.

4　结　　语

无论在岩体结构还是在土体结构中，常常遇到结构形状和荷载沿某一方向基本不变的特殊的液固耦合问题，此时如果仍采用三维有限元法来求解，难免会使问题人为地复杂化. 本文建立的广义平面应变液固耦合有限元法，可使该类问题大大简化，该方法适用于任意符合广义平面应变特征的液固耦合问题.

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2-D FEM analysis of 3-D imposed load-induced liquid-solid coupling problems

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Abstract：Based on the properties of the generalized plane strain problem，in which the total variation of all variables in a certain direction is equal to zero，a 2-D FEM with incremental variables was developed for the liquid-solid coupling problem under 3-D imposed loads. In the analysis，it was assumed that the medium was continuous，anisotropic，and saturated，Darcy's law held valid and the effective stress principle held true.

Key words：liquid-solid coupling；3-D load；plane strain；FEM